

## A New Approach for Estimating Kinetic Luminosity of Jet in AGNs \*

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**Abstract** The Königl inhomogeneous jet model can successfully reproduce most observational features of jets in active galactic nuclei (AGN), when suitable physical parameters are adopted. We improve Königl's calculations on the core emission from the jet with a small viewing angle  $\theta_0 \sim \varphi$  ( $\varphi$  is half opening angle of the conical jet). The proper motion of the jet component provides a constraint on the jet kinematics. Based on the inhomogeneous jet model, we use the proper motion data of the jet component to calculate the minimal kinetic luminosity of the jet required to reproduce the core emission measured by the very-long-baseline interferometry (VLBI) for a sample of BL Lac objects. Our results show that the minimal kinetic luminosity is slightly higher than the bolometric luminosity for most sources in the sample, which implies that radiatively inefficient accretion flows (RIAFs) may be in those BL Lac objects, or/and the properties of their broad-line regions (BLRs) are significantly different from flat-spectrum radio-loud quasars.

**Key words:** galaxies: jet — kinematics and dynamics: theory — radiation mechanisms: nonthermal—BL Lac objects: general

### 1 INTRODUCTION

Only a small fraction ( $\sim 10\%$ ) of active galactic nuclei (AGNs) are radio-loud (RL). Relativistic jets have been observed in many RL AGNs, which are believed to form very close to black holes. In the currently most favored models of jet formation, i.e., the Blandford-Payne and Blandford-Znajek mechanisms (BP and BZ, hereafter, Blandford & Payne 1982; Blandford & Znajek 1977), the power is generated through accretion and then extracted from the disk/black hole rotational energy and converted into the kinetic power of the jet. Both of these jet-formation mechanisms predict a link between the accretion disk and the jet (e.g., Cao 2002). The disk-jet connection, which indicates an intrinsic link between accretion disks and jets, has been explored by many authors using observational data in different ways (e.g., Rawlings & Saunders 1991; Xu et al. 1999; Cao & Jiang 1999, 2001; Wang et al. 2004; Liu et al. 2006). However, it is still unclear which mechanism (BP or BZ) is responsible for the jet formation in RL AGNs.

For a few sources, the kinetic power of the jets can be estimated from the X-ray observations. Young et al. (2002) derived the temperature and pressure of the gases in its inner radio lobes ( $\leq 1.5$  kpc) of M87 from the Chandra X-ray observation. The cavity of the X-ray gas is assumed to be created by the jet, and its age is estimated from the free-fall timescale. Thus, the average kinetic jet power can be estimated. The jet power can also be derived from the extended radio emission (e.g. Rawlings & Saunders 1991; Willott et al. 1999). Punsly (2005) suggested a similar relation between jet power and extended radio emission in a different way, which can almost reproduce the Willott's result within a factor of 2 (Willott et al. 1999). However, the kinetic jet power derived either from the X-ray observation or the extended radio emission is only an average over a timescale of  $\sim 10^7$  years.

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The kinematics of many jets has been measured by multi-epoch very-long-baseline interferometry (VLBI) observations, which provides a way to estimate the physical quantities of jets (e.g., Ghisellini et al. 1993; Jiang et al. 1998; Gopal-Krishna, Wiita & Dhurde 2006). Ghisellini et al. (1993) suggested that the Doppler factor of the jet component can be estimated from the VLBI and X-ray observations assuming synchrotron self-Compton radiation (SSC) to be responsible for the X-ray emission from the homogenous spherical jet component. Using this method, some important physical quantities of the jet, such as, the Doppler factor  $\delta_j$ , and the electron density  $n_e$ , can be derived from the observations. The kinetic luminosities of the jets for a sample of RL AGNs were estimated by Celotti et al. (1997). The kinetic luminosities derived in this way are found roughly consistent with (in statistic sense) the jet power estimated from the lobe emission (Rawlings & Saunders 1991).

Besides the kinetic luminosity of jet, one has to estimate radiation from the disk in order to explore the disk-jet connection in RL AGNs. The bolometric luminosity of an AGN can be estimated by integrating its spectral energy distribution (SED), however, this is only applicable for the sources well observed in multiwave-bands. For most AGNs, their bolometric luminosities are estimated from their optical or X-ray luminosities by using the empirical relation derived from a sample of AGNs (e.g. Elvis et al. 1994). The bolometric luminosity derived in this way can be mainly attributed to black hole accretion for normal bright radio-quiet (RQ) AGNs, while the situation may be different for RL AGNs. For flat-spectrum RL AGNs, the optical continuum emission may be dominated by the beamed emission from the relativistic jets. The narrow line/broad line emissions are therefore suggested to trace the emissions from the accretion disks in those sources (e.g. Rawlings & Saunders 1991), because the narrow line region (NLR)/broad line region (BLR) is believed to be photon-ionized by the radiation from the accretion disk.

In this paper, we improve Königl's calculations on the inhomogeneous conical jet model and then estimate the minimal kinetic luminosity of the jets from the VLBI core emission for a sample of BL Lac objects. The cosmological parameters,  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_\Lambda = 0.7$ , and  $\Omega_M = 0.3$  are adopted throughout the paper.

## 2 THE KÖNIGL'S INHOMOGENEOUS JET MODEL

Königl (1981) proposed an inhomogeneous conical jet model, in which the number density of electron  $n_e(r, \gamma_e)$  in the jet is assumed to be power-law dependent of the distance  $r$  from the apex of the jet as  $n_e(r, \gamma_e) = n_1(r/r_1)^{-n} \gamma_e^{-(2\alpha+1)}$ , and the magnetic field  $B(r) = B_1(r/r_1)^{-m}$ , where  $r_1 = 1 \text{ pc}$ , and  $\alpha$  is the spectral index of the optically thin synchrotron emission. The bulk motion velocity of the jet is  $\beta_j c$ , which is assumed to be constant along the jet. We have to adopt the free jet assumption to simplify the problem, which is a conventional assumption adopted in some previous papers (e.g. Hutter & Mufson 1986). The half opening angle  $\varphi = 1/\Gamma_j$  for a free jet (Blandford & Königl 1979), where the Lorentz factor  $\Gamma_j = (1 - \beta_j^2)^{-1/2}$ . Thus, the physical quantities of a relativistic jet moving at  $\beta_j c$  are available provided the three parameters:  $\alpha$ ,  $m$ , and  $n$  are specified.

The Königl's inhomogeneous jet model can successfully reproduce most observational features of radio jets in AGNs (see Jiang et al. 1998 for detailed discussion). In this model, the central part of jet is optically thick in the radio band due to synchrotron self-absorption, which can successfully explain the flat-spectrum cores of jets measured by VLBI. This model predicts a power-law dependence of the observed core size on observing frequency:  $\theta_d \propto \nu_{\text{ob}}^{-k_m}$  (Königl 1981; Jiang et al. 1998), in which  $k_m$  is given by

$$k_m = \frac{2n + m(2\alpha + 3) - 2}{2\alpha + 5}. \quad (1)$$

The spectral index of the optically thick core emission in the radio band is (Königl 1981; Jiang et al. 1998)

$$\alpha_{\text{core}} = \frac{5}{2} - \frac{4 + m}{2k_m}. \quad (2)$$

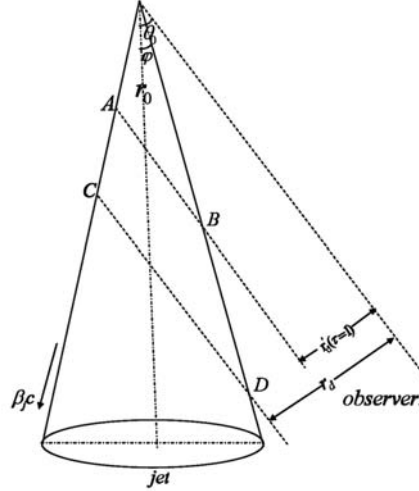
The mass conservation along the jet requires  $n = 2$ , and  $m = 1$  and  $n = 2$  are also expected by a free jet (Hutter & Mufson 1986). If  $\alpha = 0.75$ , a conventional value for the optically thin radio emission from AGN jets, is adopted, we find that  $k_m = 1$  and  $\alpha_{\text{core}} = 0$  from Equations (1) and (2). These are consistent with the observed flat-spectrum core emission and observed dependence of the core size with frequency

(i.e.,  $\theta_d \propto \nu_{\text{ob}}^{-1}$ ) in most flat-spectrum radio quasars and BL Lac objects (e.g., Padovani & Urry 1992; Padovani 1992; Jiang et al. 2001). Thus, we adopt  $\alpha = 0.75$ ,  $m = 1$  and  $n = 2$  in all our calculations.

In order to calculate the optically thick core emission from the jet, one has to calculate the optical depth of the jet for self-synchrotron absorption. In Königl (1981), the optical depth  $\tau$  is simply calculated by (see also eq. (26) in Blandford & Königl 1979)

$$\tau(\nu_{\text{ob}}, r_d) = \int k(\nu, r) dl \approx k(\nu_{\text{ob}}, r_0) \frac{2r_d \varphi}{\delta_j \sin \theta_0}, \quad (3)$$

where  $k(\nu, r)$  is the synchrotron absorption coefficient,  $\theta_0$  is the viewing angle between the jet axis and the line of sight, the Doppler factor  $\delta_j = 1/\Gamma_j(1 - \beta_j \cos \theta_0)$ , and  $\nu_{\text{ob}}$  the frequency measured in the observer's frame (see Fig. 1 for the illustration of the geometry used in the calculations). The length for the synchrotron photons travelling through the jet is appropriately considered in Equation (3) for a jet inclined at an angle of  $\theta_0$  with respect to the line of sight, while the self-synchrotron absorption coefficient is simply calculated by using the quantities at the axis of the jet with distance  $r_0$  to its apex (see Fig. 1). This, of course, is a good approximation for the case with  $\theta_0 \gg \varphi$ , as the distance to the apex of the jet changes little along the line  $AB$  in Figure 1. However, the situation would be significantly different for the case with  $\theta_0 \sim \varphi$ , for which we cannot simply adopt an average self-synchrotron absorption coefficient in the calculation of the optical depth, because the physical quantities, i.e., electron density and magnetic field strength, change significantly along the line. In this paper, we incorporate this effect in our calculation of the optical depth for the jet.



**Fig. 1** Geometry used in the calculations.

The VLBI observation usually gives the flux density and the core size. In order to compare with the observed data, we calculate the radio emission from the portion of the jet corresponding to the core size measured by the VLBI. When  $\theta_0 \sim \varphi$ , the angle between the line of sight and the direction of the bulk motion of the plasma varies from  $\theta_0 + \varphi$  to  $\theta_0 - \varphi$  (see Fig. 1). Thus, the Doppler factor is no longer a constant even if the bulk velocity is a constant along the jet. We also include this effect to integrate along the line penetrating the jet in the calculation of the optical depth, which gives

$$\begin{aligned} \tau(\nu_{\text{ob}}) = & c_2(\alpha) n_1 B_1^{(2\alpha+3)/2} [\nu_{\text{ob}}(1+z)]^{-(2\alpha+5)/2} r_1^{[2n+(2\alpha+3)m]/2} \Gamma_j^{-(2\alpha+3)/2} r_d^{-[2n+(2\alpha+3)m-2]/2} \\ & \times \int_{\theta_0-\varphi}^{\theta_0+\varphi} (1 - \beta_j \cos \theta)^{-(2\alpha+3)/2} (\sin \theta)^{[2n+(2\alpha+3)m-4]/2} d\theta, \end{aligned} \quad (4)$$

where the observed angular diameter of the core is

$$\theta_d = \frac{r_d}{D_a}, \quad (5)$$

$r_d$  is the projection of the jet portion corresponding to the observed core size  $\theta_d$ ,  $D_a$  the angular diameter distance of the source, and  $c_2(\alpha)$  a constant in the synchrotron absorption coefficient (Rybicki & Lightman 1979).

Most previous papers assumed that the VLBI core corresponds to the optically thick emission region of the jet for self-synchrotron absorption (e.g. Jiang et al. 1998). However, the possibility that the intrinsic optically thick region of the jet may be smaller than the measured VLBI core cannot be ruled out, because of the limitation of the angular resolution of VLBI observations (especially for the sources at high redshifts). Even for the sources with well resolved VLBI cores, the measured core sizes may not correspond to the surface with  $\tau = 1$ , because the core sizes are measured on the assumption of a Gaussian fitting to the emission region. In this paper, we will not be involved in the complexity of the VLBI measurements. We simply calculate the emission from the jet portion corresponding to the core size measured by VLBI, and compare it with the observed VLBI core emission. This implies that the intrinsic optically thick jet portion may be smaller than the core size measured by VLBI. The detected VLBI core emission includes the optically thin emission from the jet portion between two lines  $AB$  and  $CD$ , and the inner optically thick region with  $\tau \geq 1$  (see Fig. 1). Thus, the measured VLBI core emission consists of the emission from both the optically thick and thin regions. Letting  $\tau(\nu_{\text{ob}}) = 1$ ,  $r'_d(\tau = 1)$  can be calculated with Equation (4). The measured VLBI core flux density is

$$f_{\nu_{\text{ob}}} = f_{\nu_{\text{ob}}}(\tau \geq 1) + f_{\nu_{\text{ob}}}(\tau < 1), \quad (6)$$

where

$$f_{\nu_{\text{ob}}}(\tau \geq 1) = \frac{1}{4\pi D_a^2} \frac{c_1(\alpha)}{c_2(\alpha)} \left( \frac{\delta_j}{1+z} \right)^{1/2} B_1^{-1/2} r_1^{-m/2} \nu_{\text{ob}}^{5/2} \frac{4\varphi}{(m+4) \sin \theta_0} \frac{r'_d(\tau = 1)^{(m+4)/2}}{[\sin(\theta_0 + \varphi)]^{m/2}}, \quad (7)$$

is the flux density from the optical thick region of the jet, and

$$f_{\nu_{\text{ob}}}(\tau < 1) = \frac{r_1^3}{4D_a^2} \frac{1}{n + m(\alpha + 1) - 3} (1+z)^{-(3+\alpha)} \delta_j^{2+\alpha} c_1(\alpha) n_1 B_1^{\alpha+1} \varphi^2 \nu_{\text{ob}}^{-\alpha} \left\{ \left[ \frac{r'_d(\tau = 1)}{r_1 \sin(\theta_0 + \varphi)} \right]^{-n-m(\alpha+1)+3} - \left[ \frac{r_d}{r_1 \sin(\theta_0 + \varphi)} \right]^{-n-m(\alpha+1)+3} \right\}, \quad (8)$$

is the flux density from the optically thin region of the jet (the jet portion between  $AB$  and  $CD$  in Fig. 1). Our calculations show that the measured VLBI core emission is dominated by the emission from the optically thick portion of the jet, which implies the final results depend insensitively on the measured VLBI core sizes.

### 3 THE KINETIC LUMINOSITY $L_{\text{KIN}}$ OF THE JET

The jet can either be composed of electron-positron or electron-proton plasmas (see Worrall & Birkinshaw 2004, for a review). The detection of circularly polarized radio emission from the jets in some sources strongly suggests that the jets are composed of electron-positron plasmas with lower energy cutoff  $\gamma_{e,\text{min}} \lesssim 20$  on the pc scale (Wardle et al. 1998; Worrall & Birkinshaw 2004). However, Tavecchio et al. (2000) and Maraschi et al. (2003) used high quality broadband X-ray data to argue that the contribution of protons is necessary to explain the transport of energy to large distance. Celotti et al. (1993) suggested that  $\gamma_{e,\text{min}} \sim 1$  is required for electron-positron jets, while  $\gamma_{e,\text{min}} \simeq 100$  is required for electron-proton jets. If this is the case, the derived kinetic luminosity is almost independent of the jet composition.

We calculate the cases for electron-proton jets with  $\gamma_{e,\text{min}} \simeq 100$ , of which the results are the same for electron-positron jets with  $\gamma_{e,\text{min}} \sim 1$  (Celotti et al. 1993). The kinetic luminosity of the jet is given by

$$L_{\text{kin}} = S(u_e + u_p) \Gamma_j (\Gamma_j - 1) \beta_j c, \quad (9)$$

where  $S = 2\pi r^2(1 - \cos \varphi)$  is the cross-section of the jet,  $u_e$  and  $u_p$  are the electron and proton energy densities, respectively. The electron energy density is

$$u_e = \int_{\gamma_{e,\min}}^{\gamma_{e,\max}} n_1 \left(\frac{r}{r_1}\right)^{-n} \gamma_e^{-(2\alpha+1)} \gamma_e m_e c^2 d\gamma_e = n_1 \left(\frac{r}{r_1}\right)^{-n} m_e c^2 \frac{1}{2\alpha-1} (\gamma_{e,\min}^{1-2\alpha} - \gamma_{e,\max}^{1-2\alpha})$$

$$\approx n_1 \left(\frac{r}{r_1}\right)^{-n} m_e c^2 \frac{1}{2\alpha-1} \gamma_{e,\min}^{1-2\alpha}, \quad (10)$$

where  $\gamma_{e,\max} (\gg \gamma_{e,\min})$  is the higher energy cut-off of the electrons. The proton energy density is given by

$$u_p = \int_{\gamma_{e,\min}}^{\gamma_{e,\max}} n_1 \left(\frac{r}{r_1}\right)^{-n} \gamma_e^{-(2\alpha+1)} m_p c^2 d\gamma_e = n_1 \left(\frac{r}{r_1}\right)^{-n} m_p c^2 \frac{1}{2\alpha} (\gamma_{e,\min}^{-2\alpha} - \gamma_{e,\max}^{-2\alpha})$$

$$\approx n_1 \left(\frac{r}{r_1}\right)^{-n} m_p c^2 \frac{1}{2\alpha} \gamma_{e,\min}^{-2\alpha}. \quad (11)$$

Substituting Equations (10) and (11) into (9), we obtain

$$L_{\text{kin}} = 2\pi(1 - \cos \varphi) n_1 r_1^2 (2m_e \gamma_{e,\min}^{1-2\alpha} + \frac{2}{3} m_p \gamma_{e,\min}^{-2\alpha}) \Gamma_j (\Gamma_j - 1) \beta_j c^3. \quad (12)$$

It is still unclear on the magnetic field strength  $B$  in the jet. We use a parameter  $\beta_m$  to describe the ratio of the magnetic energy density to the total internal energy density of the plasma in the jet,

$$\beta_m = \frac{u_B}{u_B + u_e}, \quad (13)$$

where

$$u_B = \frac{B^2}{8\pi}. \quad (14)$$

#### 4 ESTIMATE OF THE MINIMAL KINETIC LUMINOSITY $L_{\text{KIN}}^{\text{MIN}}$ OF THE JET FROM VLBI OBSERVATIONS

As discussed in Section 3, the physical quantities of the jet, such as electron density  $n_1$ , magnetic field strength  $B_1$ , and the half opening angle  $\varphi$ , can be derived with Equations (9)–(14), when the kinetic luminosity  $L_{\text{kin}}$ , Lorentz factor  $\Gamma_j$ , and the fraction of the magnetic field strength  $\beta_m$  are specified. Given all physical quantities of the jet, we can calculate the VLBI core emission as a function of the viewing angle  $\theta_0$  as described in Section 2.

For the source with the proper motion  $\mu_{\text{app}}$  of jet components measured by multi-epoch VLBI observations, the apparent velocity of the jet component is

$$\beta_{j,\text{app}} = 15.81403 \mu_{\text{app}} \frac{d_L}{1.0 + z}, \quad (15)$$

where  $d_L$  is the luminosity distance in units of Gpc, and  $\mu_{\text{app}}$  is in units of mas year<sup>-1</sup>. The velocity of the jet is available by

$$\beta_j = \frac{\beta_{j,\text{app}}}{\sin \theta_0 + \beta_{j,\text{app}} \cos \theta_0}, \quad (16)$$

if the viewing angle  $\theta_0$  is known. There is an upper limit on the viewing angle  $\theta_0$ :

$$\theta_{0,\text{max}} = \arcsin \frac{2\beta_{j,\text{app}}}{\beta_{j,\text{app}}^2 + 1}, \quad (17)$$

for a measured  $\beta_{j,\text{app}}$ , as required by  $\beta_j < 1$ .

For the source with measured proper motion  $\beta_{j,\text{app}}$ , VLBI core emission, and core size, we can calculate the kinetic luminosity of the jet required to reproduce the observed VLBI core emission as a function of

**Table 1** Data Used from VLBI

Source	redshift	$\nu_{\text{ob}}$ (GHz)	$f_c$ (Jy)	$\theta_d$ (mas)	$\mu$ (mas year <sup>-1</sup> )	Ref
0003–066	0.347	15.0	1.850	0.173	0.010	1
0219+428	0.444	43.2	0.593	0.025	1.110	2
0235+164	0.940	43.2	0.705	0.080	0.930	3
0454+844	0.112	5.0	1.300	0.550	0.140	3
0716+714	0.300	43.2	0.390	0.040	1.200	2
0735+178	0.424	15.0	0.950	0.147	0.640	1
0754+100	0.266	15.0	1.420	0.114	0.700	1
0808+019	0.930	15.0	1.270	0.040	0.110	1
0814+425	0.258	15.0	1.080	0.063	0.320	1
0823+033	0.506	15.0	1.100	0.074	0.480	1
0829+046	0.180	22.2	0.796	0.053	1.400	2
0851+202	0.306	43.2	1.640	0.042	0.670	2
0954+658a	0.368	5.0	0.477	0.190	0.440	3
1101+384	0.031	15.0	0.450	0.106	1.330	1
1219+285	0.102	22.2	0.263	0.09	0.600	2
1308+326	0.996	15.0	2.590	0.141	0.750	1
1413+135	0.247	15.0	1.420	0.046	0.450	1
1652+398	0.034	15.0	0.540	0.219	0.960	1
1749+701	0.770	15.0	0.570	0.080	0.260	1
1749+096	0.320	15.0	5.550	0.057	0.150	1
1803+784	0.684	5.0	1.436	0.200	0.000	3
1807+698	0.051	15.0	0.830	0.125	2.600	1
1823+568	0.664	15.0	2.140	0.130	0.120	1
2007+776	0.342	5.0	1.361	0.190	0.180	3
2131–021	1.285	15.0	1.150	0.134	0.120	1
2200+420	0.069	15.0	2.960	0.105	1.410	1

Reference–1: Kovalev et al. (2005); 2: Jorstad et al. (2001); 3: Jiang et al. (2001).

viewing angle  $\theta_0$  ( $\varphi < \theta_0 < \theta_{0,\text{max}}$ ). For the source with measured  $\beta_{j,\text{app}}$ , the physical quantities of the jet are available from Equations (9)–(14) for a given kinetic luminosity  $L_{\text{kin}}$  as a function of viewing angle. Then we can calculate its core luminosity based on derived quantities of the jet with Equation (6). Tuning  $\theta_0$  from  $\varphi$  to  $\theta_{0,\text{max}}$ , the maximal core luminosity  $L_{\text{core}}^{\text{max}}$  can be derived for the given  $L_{\text{kin}}$ . Changing the value of  $L_{\text{kin}}$ , we can find  $L_{\text{kin}}^{\text{min}}$  when  $L_{\text{core}}^{\text{max}} = L_{\text{core}}^{\text{obs}}$ . Thus, the minimal jet kinetic luminosity  $L_{\text{kin}}^{\text{min}}$  required to reproduce the VLBI observations is derived.

## 5 RESULTS

Using the approach described in Section 4, we can estimate the minimal kinetic luminosity of the jet, measure the proper motion of jet component, the VLBI core emission and size. We compile a sample of 26 BL Lac objects with all necessary data for estimating the minimal jet kinetic luminosity. All the data of the sources in this sample are summarized in Table 1. The average core flux density for each source is adopted in our calculations. For six sources in our sample, we cannot find a kinetic luminosity  $L_{\text{kin}}$  that can reproduce the observed VLBI core emission (see Tab. 2). The kinematics of the jets is inferred from the measured proper motions of the jet components at  $\sim$ pc length-scales, which may be different from that of the jets near the black holes in these six sources. If the Lorentz factors of the jets near the black holes in these sources are higher than those derived from the proper motions, it is then possible for us to derive the kinetic luminosities for these sources.

Our estimates on the minimal kinetic luminosity are carried out by assuming equipartition between the magnetic and electron energy densities, i.e.,  $\beta_m = 0.5$ . If the magnetic field is weaker than the equipartition value, the derived  $L_{\text{kin}}^{\text{min}}$  should be higher than the present value. Thus, the values of  $L_{\text{kin}}^{\text{min}}$  derived in this paper are indeed the minimal kinetic luminosities. In Figure 2 we plot the core luminosity of the jet as a function of kinetic luminosity of the jet  $L_{\text{kin}}$  for the source 0219+428. Comparing with the core luminosity measured by VLBI, we can derive the minimal kinetic luminosity of the jet in this source (see Fig. 2). In this way, we can estimate the minimal kinetic luminosities for all the sources in this sample.

The VLBI data are needed in the present estimate of kinetic luminosity. Another widely used method to estimate the jet power averaged over a long timescale is from the radio emission of its lobes. An empirical

**Table 2** Derived Physical Quantities of the Sources in the Sample

Source	$\log L_{\text{kin}}^{\text{min}}(\text{erg s}^{-1})$	$\log L_{\text{bol}}(\text{erg s}^{-1})$	Ref	$\log Q_{\text{jet}}(\text{erg s}^{-1})$	Ref	$m_{\text{R}}(\text{host})$	Ref	$\log M_{\text{bh}}/M_{\odot}$
0003–066	...	44.15	1	...	..	...		...
0219+428	45.14	...	..	44.43	1	...		...
0235+164	45.60	44.86	1	43.83	1	> 19.75	2	< 10.54
0454+844	44.27	...	..	42.19	1	> 22.37	2	< 7.24
0716+714	44.73	...	..	...	..	> 20.00	2	< 7.84
0735+178	45.28	...	..	42.04	1	> 20.44	2	< 8.42
0754+100	46.36	...	..	42.14	1	> 18.69	2	< 9.03
0808+019	...	...	..	44.89	1	...		...
0814+425	...	42.81	1	42.96	1	> 21.47	2	< 6.95
0823+033	45.91	44.40	1	42.51	1	> 20.18	2	< 8.88
0829+046	46.12	...	..	42.64	1	16.94	2	8.83
0851+202	46.55	44.60	1	42.22	1	> 18.53	2	< 8.78
0954+658a	45.13	43.63	1	42.94	1	> 19.60	2	< 8.56
1101+384	44.10	42.40	1	41.62	1	...		...
1219+285	43.95	43.25	1	40.90	1	...		...
1308+326	46.11	46.12	1	44.00	1	...		...
1413+135	...	...	..	...	..	...		...
1652+398	46.65	42.36	1	41.32	1	...		...
1749+096	...	...	..	...	..	18.82	2	8.73
1749+701	46.55	...	..	41.59	1	> 19.28	2	< 10.49
1803+784	...	45.56	1	43.09	1	> 20.89	2	< 9.22
1807+698	46.12	42.40	1	42.63	1	13.87	2	8.87
1823+568	46.02	44.32	1	44.62	1	20.24	2	9.47
2007+776	45.09	...	..	42.91	1	19.03	2	8.88
2131–021	46.73	44.66	1	44.33	1	> 21.98	2	< 10.39
2200+420	44.57	43.38	1	41.67	1	15.37	2	8.71

Reference–1: Gu, Cao & Jiang (2006); 2: Urry et al. (2000).

relation between the extended radio emission and jet power,

$$Q_{\text{jet}} = 3 \times 10^{38} f^{3/2} L_{\text{ext},151}^{6/7}, \quad (18)$$

is proposed by Willott et al. (1999), where  $L_{\text{ext},151}$  is the extended radio luminosity at 151 MHz in units of  $10^{28} \text{ W Hz}^{-1} \text{ sr}^{-1}$ , and  $1 \leq f \leq 20$ . The low-frequency radio emission (e.g., 151 MHz) may still be Doppler beamed for BL Lac objects. We therefore use the extended radio emission detected at 5 GHz by VLA to estimate the jet power. The extended radio emission measured by VLA is K-corrected to 151 MHz in the rest frame of the source. We compare the derived minimal kinetic luminosity  $L_{\text{kin}}^{\text{min}}$  with the jet power estimated from the extended radio emission in Figure 3. The linear regression gives

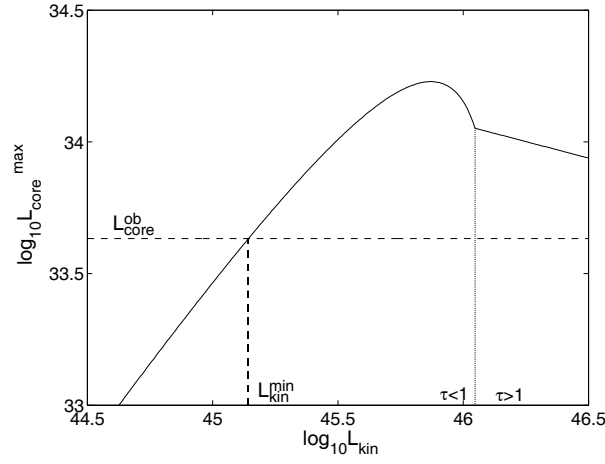
$$\log L_{\text{kin}}^{\text{min}} = 0.24(\pm 0.83) \log Q_{\text{jet}} + 35.40(\pm 1.03), \quad (19)$$

for  $f = 1$  (see Fig. 3).

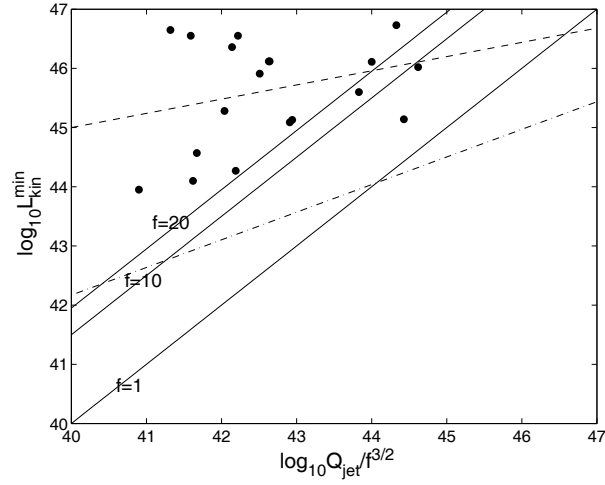
Best et al. (2006) also derived a relation between mechanical luminosity of radio sources and monochromatic 1.4-GHz radio luminosity:

$$\frac{L_{\text{mech}}}{10^{36} \text{ W}} = (3.0 \pm 0.2) \left( \frac{L_{1.4\text{GHz}}}{10^{25} \text{ W Hz}^{-1}} \right)^{0.40 \pm 0.13}, \quad (20)$$

where the mechanical luminosities  $L_{\text{mech}}$  are estimated by Bîrzan et al. (2004) from hot gases in cavities and bubbles. They estimated the hot gas pressure and the size of the cavity/bubble from the X-ray observations to calculate the total paper done by the jet with  $\sim pdV$ . The cavity age is estimated as the time required for the cavity to rise the projected distance from the radio core to its present location at the speed of sound, then the mechanical luminosity  $L_{\text{mech}}$  is derived, which should be comparable with the kinetic jet luminosity derived in this paper, except that it is an average over  $\gtrsim 10^7$  years and our estimate is nearly a real-time one.



**Fig. 2** Core luminosity  $L_{\text{core}}^{\text{max}}$  as a function of  $L_{\text{kin}}$  for the BL Lac object 0219+428. The dotted line represents  $\tau = 1$  corresponding to observed VLBI core size.



**Fig. 3** Relation between  $L_{\text{kin}}^{\text{min}}$  and  $Q_{\text{jet}}$ . The different solid lines correspond to  $Q_{\text{jet}} = L_{\text{kin}}^{\text{min}}$  for different values of  $f$  adopted in the Eq. (18); The dashed line shows the linear regression for the sample. The dash-dotted line shows the relation between  $L_{\text{mech}}$  and  $Q_{\text{jet}}$  given by Best et al. (2006).

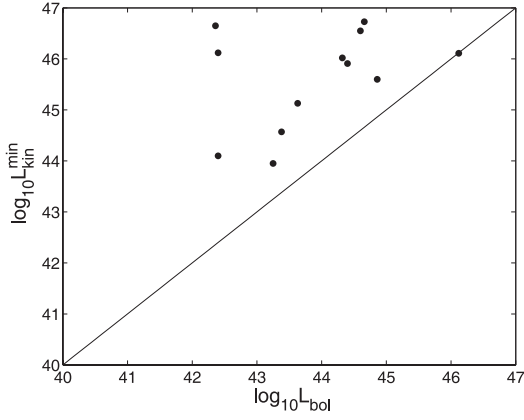
For BL Lac objects, the continuum emission may probably be dominated by the beamed jet emission. We use the broad-line emission to obtain their bolometric luminosity. We estimate the luminosity in broad emission line  $L_{\text{BLR}}$  with the same way as Celotti et al. (1997). An empirical relation  $L_{\text{bol}} \sim 10L_{\text{BLR}}$  is used to estimate their bolometric luminosities (Elvis et al. 1994).

McLure & Dunlop (2004) derived a tight correlation between host galaxy absolute magnitude  $M_{\text{R}}$  at R-band and the central black hole mass:

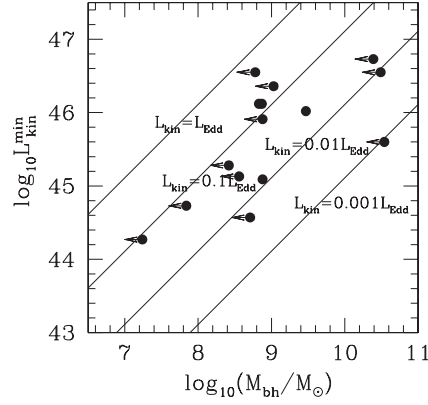
$$\log_{10}(M_{\text{bh}}/M_{\odot}) = -0.50(\pm 0.02)M_{\text{R}} - 2.75(\pm 0.53). \quad (21)$$

The central black hole masses  $M_{\text{bh}}$  in the sources of this sample can be estimated from their host galaxy luminosities with Equation (21). We search the literature and find host galaxy absolute magnitudes for 18 sources in our sample, which are listed in Table 2.





**Fig. 4** Relation between  $L_{\text{kin}}^{\text{min}}$  and  $L_{\text{bol}}$ . The line represents  $L_{\text{kin}}^{\text{min}} = L_{\text{bol}}$ .



**Fig. 5** Relation between  $L_{\text{kin}}^{\text{min}}$  and  $M_{\text{bh}}$ .

The relation between  $L_{\text{kin}}^{\text{min}}$  and  $L_{\text{bol}}$  is plotted in Figure 4, and we find that  $L_{\text{bol}} \lesssim L_{\text{kin}}^{\text{min}}$  for most sources. We plot the relation between  $L_{\text{kin}}^{\text{min}}$  and  $M_{\text{bh}}$  in Figure 5, and find most sources have  $L_{\text{kin}}^{\text{min}}/L_{\text{Edd}} \lesssim 0.1$ .

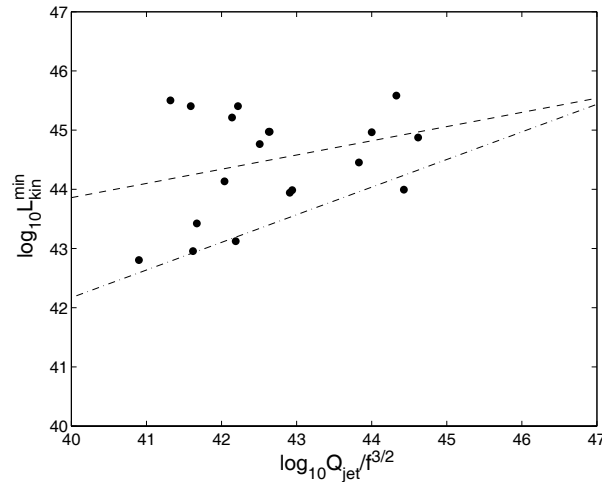
## 6 DISCUSSION

In this paper, we propose an approach to estimate the minimal kinetic luminosity from the VLBI core emission and the proper motion of the jet component measured by the VLBI based on the Königl's inhomogeneous jet model. The derived minimal jet kinetic luminosities  $L_{\text{kin}}^{\text{min}}$  for these BL Lac objects are systematically higher than the jet power estimated from the extended radio emission (Fig. 3). The minimal kinetic luminosity always requires equipartition between magnetic field and electron energy density in order to reproduce observed VLBI core emission. This can be naturally understood, because the core emission increases with the magnetic field strength  $B$ , if all other parameters of the jet are fixed. We adopt a set of parameters, which can well reproduce the main features of VLBI observations on the jets in AGNs (Sect. 2). We have also performed calculations based on different sets of jet parameters, and find that our results on  $L_{\text{kin}}$  are always insensitive to the adopted jet parameters.

The jet power estimated from the extended radio emission is an average over a timescale of  $\sim 10^6 - 10^7$  years. Our present estimate on the jet kinetic luminosity is based on the emission from the core with size  $\lesssim 1\text{pc}$ , which is almost the real time value of the jet power. It is obviously a better choice to use the kinetic luminosity of jet, instead of the jet power estimated from the extended radio emission, to explore the physics of disk-jet connection in blazars.

We find that the slopes are similar for  $Q_{\text{jet}} - L_{\text{kin}}^{\text{min}}$  and  $L_{\text{bol}} - L_{\text{kin}}^{\text{min}}$  (see Figs. 3 and 4), though only lower limits on the kinetic luminosities are derived in this paper. The bolometric luminosity is estimated from the BLR luminosity for these sources, because the continuum emission is usually dominated by the beamed jet emission. Nearly linear correlations between extended radio emission and emission line luminosity have been found for radio quasars/BL Lac objects and FRI radio galaxies, respectively (see Cao & Jiang 2001; Cao & Rawlings 2004), which may be the reason for the similar slopes found in Figures 3 and 4.

Our estimate of  $L_{\text{kin}}^{\text{min}}$  depends systematically on the lower energy cutoff for electrons adopted (see Eq. (12)). Compared with the relation between radio luminosity and mechanical luminosity  $L_{\text{mech}}$  derived by Best et al. (2006), the relation between  $Q_{\text{jet}}$  and  $L_{\text{kin}}^{\text{min}}$  we derived has a similar slope as theirs, but at roughly more than an order of magnitude higher than theirs (see Fig. 3). The reason causing such a discrepancy is still unclear. We adopt the lower energy cutoff of electrons  $\gamma_{e,\text{min}} \sim 100$  in all calculations as done by Celotti & Fabian (1993). However, a simple one-zone synchrotron+Inverse-Compton model (Tavecchio et al. 2000) can reproduce the observed radio, optical, and X-ray fluxes of some powerful radio quasars quite well, if  $\gamma_{e,\text{min}} \sim 500 - 1000$  is adopted (Tavecchio et al. 2005). If this is the case also for



**Fig. 6** As same as Fig. 3, but  $L_{\text{kin}}^{\text{min}}$  is derived by assuming a lower energy cutoff of electrons  $\gamma_{\text{e,min}} = 1000$ .

BL Lac objects, the discrepancy can be alleviated. If  $\gamma_{\text{e,min}} = 1000$  is adopted, the value of  $L_{\text{kin}}^{\text{min}}$  would be systematically lower, i.e.,  $\sim 1/30$  of that derived for  $\gamma_{\text{e,min}} = 100$ . In Figure 6 we plot the relation between jet power  $Q_{\text{jet}}$  and  $L_{\text{kin}}^{\text{min}}$ , where the minimal kinetic luminosities  $L_{\text{kin}}^{\text{min}}$  are derived by assuming  $\gamma_{\text{e,min}} = 1000$  (instead of  $\gamma_{\text{e,min}} = 100$  in Fig. 3). We find that our results are roughly consistent with that derived by Best et al. (2006). This can be tested by detailed spectral model calculations similar to those done by Tavecchio et al. (2005) on these BL Lac objects, which is beyond the scope of this paper. In principle, this can also be tested if the present sample is enlarged to quasars. Our present calculations show that the derived minimal kinetic luminosities are usually corresponding to  $\theta_0 \simeq \varphi$ , which could be a fairly good approximation for BL Lac objects. However, it may not be suitable for quasars, because the viewing angles for most radio-loud quasars may be larger than those for BL Lac objects.

For a jet magnetically accelerated either from a rapidly rotating black hole or a standard accretion disk, its kinetic luminosity  $L_{\text{kin}}$  is usually required to be lower than the disk luminosity (e.g., Livio et al. 1999; Cao 2004). We find that most BL Lac objects in our sample have  $L_{\text{kin}}^{\text{min}} \gtrsim L_{\text{bol}}$  (see Fig. 4), which implies that the accretion disks of the BL Lac objects should be radiatively inefficient (e.g., Narayan & Yi 1995; Cao 2003). In this case, the gravitational energy of the accretion flows can sufficiently power the jets, but only a small fraction of the gravitational energy released is radiated away. Thus, it can be understood that  $L_{\text{kin}} > L_{\text{bol}}$  for most BL Lac objects in this sample. In Figure 5 we find most sources in these sample have  $L_{\text{kin}}^{\text{min}} \lesssim 0.1L_{\text{Edd}}$  (or  $L_{\text{kin}}^{\text{min}} \lesssim 0.01L_{\text{Edd}}$ , if  $\gamma_{\text{e,min}} = 1000$  is adopted), which implies that they are indeed accreting at relatively low rates, as required by RIAF/ADAF models (e.g. Narayan & Yi 1995). It should be cautious on the sources with  $L_{\text{kin}}^{\text{min}} > L_{\text{bol}}$ , as we estimate the bolometric luminosity  $L_{\text{bol}}$  from the BLR luminosity using an empirical relation  $L_{\text{bol}} = 10L_{\text{BLR}}$ , which is derived from normal quasars. The RIAFs (ADAFs) may be present in the BL Lac objects. It is well known that the RIAFs have relatively lower radiative efficiencies than standard thin disks, and their SEDs are quite different from those of normal quasars, so it is unclear whether  $L_{\text{bol}} = 10L_{\text{BLR}}$  still holds for BL Lac objects. We cannot rule out the possibility that the BLRs in BL Lac objects may be systematically different from those in quasars.

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## References

- Best P. N., Kaiser C. R., Heckman T. M., Kauffmann G., 2006, MNRAS, 368, 67
- Birzan L., Rafferty D. A., McNamara B. R. et al., 2004, ApJ, 607, 800
- Blandford R. D., Königl A., 1979, ApJ, 232, 34
- Blandford R. D., Payne D. G., 1982, MNRAS, 199, 883
- Blandford R. D., Znajek R. L., 1977, MNRAS, 179, 433
- Cao X., 2002, MNRAS, 332, 999
- Cao X., 2003, ApJ, 599, 147
- Cao X., 2004, ApJ, 613, 716
- Cao X., Jiang D. R., 1999, MNRAS, 307, 802
- Cao X., Jiang D. R., 2001, MNRAS, 320, 347
- Cao X., Rawlings S., 2004, MNRAS, 349, 1419
- Celotti A., Fabian A. C., 1993, MNRAS, 264, 228
- Celotti A., Padovani P., Ghisellini G., 1997, ApJ, 480, 534
- Elvis M. et al., ApJS, 1994, 95, 1
- Ghisellini G., Padovani P., Celotti A., Maraschi L., 1993, ApJ, 407, 65
- Gopal-Krishna Wiita P. J., Dhurde S., 2006, MNRAS, 369, 1287
- Hutter D. J., Mufson S. L., 1986, ApJ, 301, 50
- Jiang D. R., Cao X., Hong X., 1998, ApJ, 494, 139
- Jiang D. R., Cao X. W., Zhou J. F., In: Galaxies and their Constituents at the Highest Angular Resolutions, Proceedings of IAU Symposium 205, held 15–18 August 2000 at Manchester, United Kingdom, R. T. Schilizzi, ed., 2001, p.148
- Jorstad S. G., Marscher A. P., Mattox J. R. et al., 2001, ApJS, 134, 181
- Königl A., 1981, ApJ, 243, 700
- Kovalev Y. Y., Kellermann K. I., Lister M. L. et al., 2005, AJ, 130, 2473
- Liu Y., Jiang D. R., Gu M. F., 2006, ApJ, 637, 669
- Livio M., Ogilvie G. I., Pringle J. E., 1999, ApJ, 512, 100
- Maraschi L., Tavecchio F., 2003, ApJ, 593, 667
- McLure R., Jarvis M. J., 2004, MNRAS, 352, 1390
- Narayan R., Yi I., 1995, ApJ, 452, 710
- Padovani P., 1992, A&A, 256, 399
- Padovani P., Urry C. M., 1992, ApJ, 387, 449
- Punsly B., 2005, ApJ, 623, L9
- Rawlings S., Saunders R., 1991, Nature, 349, 138
- Rybicki G. B., Lightman A. P., 1979, Radiative processes in astrophysics, New York, Wiley-Interscience.
- Tavecchio F. et al., 2005, ApJ, 630, 721
- Tavecchio F. et al., 2000, ApJ, 543, 535
- Urry C. M. et al., 2000, ApJ, 532, 816
- Wang J.-M., Luo B., Ho L. C., 2004, ApJ, 615, L9
- Wardle J. F. C. et al., 1998, Nature, 395, 457
- Willott C. J., Rawlings S., Blundell K. M., Lacy M., 1999, MNRAS, 309, 1017
- Worrall D. M., Birkinshaw M., 2006, In: Physics of Active Galactic Nuclei at all Scales, D. Alloin, R. Johnson, P. Lira, eds., Lecture Notes in Physics, Vol. 693, Springer, Berlin and Heidelberg, p.39
- Xu C., Livio M., 1999, AJ, 118, 1169
- Young A. J., Wilson A. S., Mundell C. G., 2002, ApJ, 579, 560